Oxford Cambridge and RSA

Wednesday 22 June 2022 – Afternoon

A Level Further Mathematics B (MEI)

Y434/01 Numerical Methods

Time allowed: 1 hour 15 minutes

* 9 4 5 3 5 7 4 8 1 8 *

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **12** pages.

ADVICE
- Read each question carefully before you start your answer.

Answer **all** the questions.

1 $C = 3.7622$ and $S = 3.6269$ are used to approximate $\cosh 2$ and $\sinh 2$ respectively.

(a) Determine whether these approximations are the result of chopping or rounding the values of $\cosh 2$ and $\sinh 2$. **[1]**

(b) Calculate the relative error when $C^2 - S^2$ is used to approximate $\cosh^2 2 - \sinh^2 2$, giving your answer correct to **3** significant figures. **[2]**

(c) **Without** doing any further calculations, explain whether the same value for the relative error is obtained when $(C-S)^2$ is used to approximate $(\cosh 2 - \sinh 2)^2$. **[1]**

2 The table shows some values of x and the associated values of $y = \mathrm{f}(x)$.

x	2.75	3	3.25
$\mathrm{f}(x)$	0.920799	1	1.072858

(a) Calculate an estimate of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $x = 3$ using the forward difference method, giving your answer correct to **5** decimal places. **[2]**

(b) Calculate an estimate of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $x = 3$ using the central difference method, giving your answer correct to **5** decimal places. **[2]**

(c) Explain why your answer to part **(b)** is likely to be closer than your answer to part **(a)** to the true value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $x = 3$. **[1]**

When $x = 5$ it is given that $y = 1.4645$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = 0.1820$, correct to **4** decimal places.

(d) Determine an estimate of the error when $\mathrm{f}(5)$ is used to estimate $\mathrm{f}(5.024)$. **[2]**

3 The equation $f(x) = \sin^{-1}(x) - x + 0.1 = 0$ has a root α such that $-1 < \alpha < 0$.

Alex uses an iterative method to find a sequence of approximations to α. Some of the associated spreadsheet output is shown in the table.

	C	D	E
4	r	x_r	$f(x_r)$
5	0	−1	−0.4707963
6	1	−0.8	−0.0272952
7	2	−0.787691	−0.0193610
8	3	−0.7576546	−0.0020574
9	4	−0.7540834	−0.0001740
10	5		
11	6		

The formula in cell D7 is

```
=(D5*E6−D6*E5)/(E6−E5)
```

and equivalent formulae are in cells D8 and D9.

(a) State the method being used. **[1]**

(b) Use the values in the spreadsheet to calculate x_5 and x_6, giving your answers correct to **7** decimal places. **[3]**

(c) State the value of α as accurately as you can, justifying the precision quoted. **[1]**

Alex uses a calculator to check the value in cell D9, his result is -0.7540832686.

(d) Explain why this is different to the value displayed in cell D9. **[1]**

The value displayed in cell E11 in Alex's spreadsheet is −1.4629E-09.

(e) Write this value in standard mathematical notation. **[1]**

4 **Fig. 4.1** shows part of the graph of $y = e^x - x^2 - x - 1.1$.

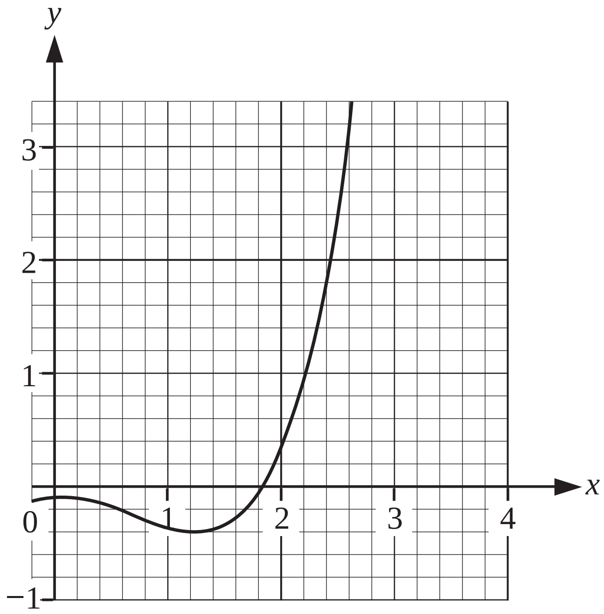

Fig. 4.1

The equation $e^x - x^2 - x - 1.1 = 0$ has a root α such that $1 < \alpha < 2$.

Ali is considering using the Newton-Raphson method to find α. Ali could use a starting value of $x_0 = 1$ or a starting value of $x_0 = 2$.

(a) **Without** doing any calculations, explain whether Ali should use a starting value of $x_0 = 1$ or a starting value of $x_0 = 2$, or whether using either starting value would work equally well. **[2]**

Ali is also considering using the method of fixed point iteration to find α. Ali could use a starting value of $x_0 = 1$ or a starting value of $x_0 = 2$.

Fig. 4.2 shows parts of the graphs of $y = x$ and $y = \ln(x^2 + x + 1.1)$.

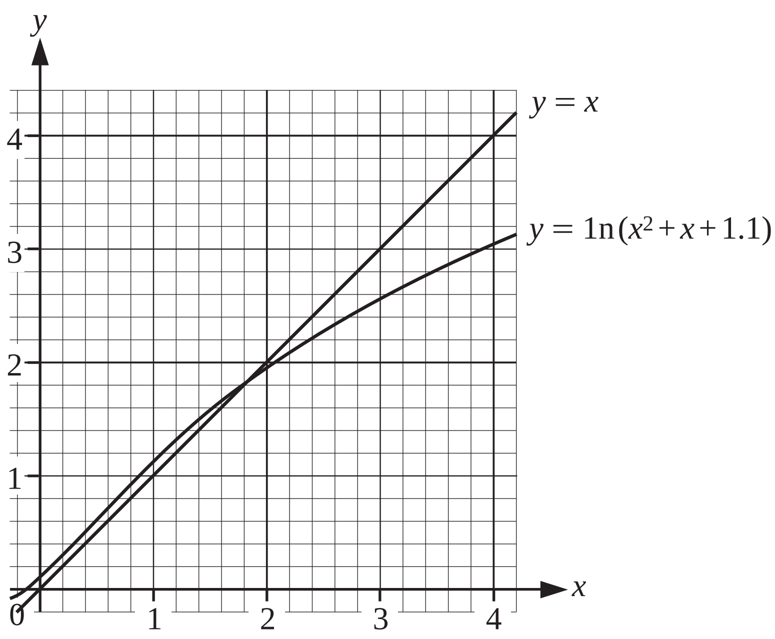

Fig. 4.2

(b) **Without** doing any calculations, explain whether Ali should use a starting value of $x_0 = 1$ or a starting value of $x_0 = 2$ or whether either starting value would work equally well. **[2]**

Ali used one of the above methods to find a sequence of approximations to α. These are shown, together with some further analysis in the associated spreadsheet output in **Fig. 4.3**.

		M	N	O
	r	x_r		
4	0	2		
5	1	1.879008	−0.121	
6	2	1.858143	−0.021	0.172
7	3	1.857565	−6E-04	0.028
8	4	1.857564	−4E-07	8E-04
9	5	1.857564	−2E-13	6E-07

Fig. 4.3

The formula in cell N5 is =M5−M4

and the formula in cell O6 is =N6/N5

equivalent formulae are in cells N6 to N9 and O7 to O9 respectively.

(c) State what is being calculated in the following columns of the spreadsheet.

(i) Column N **[1]**

(ii) Column O **[1]**

(d) Explain whether the values in column O suggest that Ali used the Newton-Raphson method or the iterative formula $x_{n+1} = \ln\left(x_n^2 + x_n + 1.1\right)$ to find this sequence of approximations to α. **[2]**

5 Kai uses the midpoint rule, trapezium rule and Simpson's rule to find approximations to $\int_a^b \mathrm{f}(x)\,\mathrm{d}x$, where a and b are constants. The associated spreadsheet output is shown in the table. Some of the values are missing.

	F	G	H	I
3	n	M_n	T_n	S_{2n}
4	1	0.2436699	0.1479020	
5	2	0.2306967		

(a) Write down a suitable spreadsheet formula for cell H5. **[2]**

(b) Complete the copy of the table in the Printed Answer Booklet, giving the values correct to **7** decimal places. **[4]**

(c) Use your answers to part **(b)** to determine the value of $\int_a^b \mathrm{f}(x)\,\mathrm{d}x$ as accurately as you can, justifying the precision quoted. **[3]**

6 Charlie uses fixed point iteration to find a sequence of approximations to the root of the equation

$\sin^{-1}(x) - x^2 + 1 = 0$.

Charlie uses the iterative formula $x_{n+1} = \mathrm{g}(x_n)$, where $\mathrm{g}(x_n) = \sin\left(x_n^2 - 1\right)$.
Two sections of the associated spreadsheet output, showing x_0 to x_6 and x_{102} to x_{108}, are shown in **Fig. 6.1**.

r	x_r	difference	ratio
0	0		
1	−0.841471	−0.84147	
2	−0.287798	0.553673	−0.65798
3	−0.793885	−0.50609	−0.91405
4	−0.361379	0.432507	−0.85461
5	−0.763945	−0.40257	−0.93078
6	−0.404459	0.359486	−0.89299

102	−0.596302	0.004626	−0.95886
103	−0.600738	−0.00444	−0.95911
104	−0.596484	0.004254	−0.95887
105	−0.600564	−0.00408	−0.95910
106	−0.596652	0.003912	−0.95888
107	−0.600404	−0.00375	−0.95909
108	−0.596806	0.003598	−0.95889

Fig. 6.1

(a) Use the information in **Fig. 6.1** to find the value of the root as accurately as you can, justifying the precision quoted. **[4]**

The relaxed iteration $x_{n+1} = (1-\lambda)x_n + \lambda g(x_n)$, with $\lambda = 0.51$ and $x_0 = 0$, is to be used to find the root of the equation $\sin^{-1}(x) - x^2 + 1 = 0$.

(b) Complete the copy of **Fig. 6.2** in the Printed Answer Booklet, giving the values of x_r correct to **7** decimal places and the values in the difference column and ratio column correct to **3** significant figures.

r	x_r	difference	ratio
0	0		
1			
2			
3			
4		−0.000192	
5		-1.99×10^{-7}	0.00103
6		-1.82×10^{-10}	0.000914

Fig. 6.2

[4]

(c) Write down the value of the root correct to **7** decimal places. **[1]**

(d) Explain why extrapolation could not be used in this case to find an improved approximation using this sequence of iterates. **[1]**

In this case the method of relaxation has been used to speed up the convergence of an iterative scheme.

(e) Name another application of the method of relaxation. **[1]**

7 Sam decided to go on a high-protein diet. Sam's mass in kg, M, after t days of following the diet is recorded in **Fig. 7.1**.

t	0	10	20	30
M	88.3	80.05	78.7	78.85

Fig. 7.1

A difference table for the data is shown in **Fig. 7.2**.

t	M	ΔM	$\Delta^2 M$	$\Delta^3 M$
0	88.3			
10	80.05			
20	78.7			
30	78.85			

Fig. 7.2

(a) Complete the copy of the difference table in the Printed Answer Booklet. **[1]**

Sam's doctor uses these data to construct a cubic interpolating polynomial to model Sam's mass at time t days after starting the diet.

(b) Find the model in the form $M = at^3 + bt^2 + ct + d$, where a, b, c and d are constants to be determined. **[4]**

Subsequently it is found that when $t = 40$, $M = 78.7$ and when $t = 50$, $M = 80.05$.

(c) Determine whether the model is a good fit for these data. **[2]**

(d) By completing the extended copy of **Fig. 7.2** in the Printed Answer Booklet, explain why a quartic model may be more appropriate for the data. **[2]**

(e) Refine the doctor's model to include a quartic term. **[3]**

(f) Explain whether the new model for Sam's mass is likely to be appropriate over a longer period of time. **[2]**

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